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Flattening Single-Vertex Origami: The Non-Expansive Case

Very little is known about the foldability and reconfiguration of rigid origami: flat pieces of paper creased along internal edges forming a plane graph, allowed to bend along the creases while maintaining the rigid geometry of the faces. But no progress can be made without fully understanding the simplest case, which appears as a necessary step in any possible method: the single-vertex origami. For a vertex placed in the interior of the piece of paper, or for a boundary vertex incident to a paper angle of at most π, Streinu and Whiteley solved the problem by reducing it to the Carpenter's Rule Problem for closed spherical polygons. Their method, using expansive motions induced by pointed spherical pseudo-triangulation mechanisms, can be applied to unfold closed spherical polygons of total length less than 2π, and open ones less than π. It is known that lengths larger than 2π may not be reconfigurable.

The remaining case, open polygons of length between π and 2π, is not directly amenable to the pseudo-triangulation technique, as it requires both contractive and expansive motions. Here, we settle the problem by showing that it is always possible to unfold, without self-collisions, a spherical bar-and-joint polygonal path of total length between π and 2π. The motion (necessarily partially non-expansive) can be carried out in discrete steps, and completed in finite time, for which we give precise bounds.